Model specification

This document provides a brief description of the Markov-Switching model for the Conference Board's Composite Coincident and Leading Indexes [CCI and CLI] from which recession probability forecasts are obtained. For further details we refer to Paap *et al.* (2008).

Non-technical description

The model is built on the assumption that the business cycle behavior of both variables is characterized by different mean growth rates in 'recession' and 'expansion' regimes. The business cycle regimes are not imposed exogenously, but are incorporated endogenously in the model by means of a latent two-state Markov process. The idea for modeling business cycle regimes in this way was introduced into the econometrics literature by Hamilton (1989). In the twenty years since this seminal article was published, the Markov-Switching model has been applied successfully for describing the typical business cycle behavior of macroeconomic variables such as output and employment, and for turning point forecasting, see Chauvet and Hamilton (2006) and Chauvet and Piger (2008) for recent examples.

Our model is a bivariate Markov-Switching model for the monthly growth rates of the Conference Board's Composite Coincident and Leading Indexes [CCI and CLI]. These two variables are assumed to share the same business cycle, which is achieved by having only a single Markov process governing the regime switches of both variables in the model. Obviously, the cycle of the CLI leads the cycle of the CCI, which is accounted for in the model. Furthermore, stylized facts show that on average the CLI has a longer lead time when entering a recession than when entering an expansion (The Conference Board, 2001). This characteristic is also incorporated in the model. A final feature of the model is that it accounts for the large and persistent decline in volatility that many US macro-economic variables such as output have shown since the mid-1980s, see McConnell and Perez-Quiros (2000) and Sensier and van Dijk (2004), among others. To accommodate the effects of this 'Great Moderation' we allow for a single structural change in the variances and covariance of the shocks to the CCI and CLI.

Technical description

Let $y_{1,t}$ and $y_{2,t}$ denote the growth rates of the Conference Board's Composite Coincident and Leading Index [CCI and CCI], respectively, in month t. The model is built on the assumption that the business cycle behavior of both variables is characterized by different mean growth rates in 'recession' and 'expansion' regimes. This is formalized by defining the unobserved binary random variables $s_{1,t}$ and $s_{2,t}$, where $s_{j,t}$ takes the value 0 in case $y_{j,t}$ is in expansion and 1 in case $y_{j,t}$ is in the recession regime. The mean growth rate conditional on the state $s_{j,t}$ is denoted as $\mu_{j,s_{j,t}} \equiv \mathsf{E}[y_{j,t}|s_{j,t}]$, for $j = 1, 2$, where typically $\mu_{j,1} < 0 < \mu_{j,0}$ such that recessions and expansions correspond to periods with negative and positive average growth, respectively. Finally, assuming first-order autoregressive dynamics in the demeaned growth rates, we arrive at the specification

$$\begin{aligned} y_{1,t} - \mu_{1,s_{1,t}} &= \phi_{1,1}(y_{1,t-1} - \mu_{1,s_{1,t-1}}) + \phi_{1,2}(y_{2,t-1} - \mu_{2,s_{2,t-1}}) + \varepsilon_{1,t} \\ y_{2,t} - \mu_{2,s_{2,t}} &= \phi_{2,1}(y_{1,t-1} - \mu_{1,s_{1,t-1}}) + \phi_{2,2}(y_{2,t-1} - \mu_{2,s_{2,t-1}}) + \varepsilon_{2,t}, \end{aligned} \tag{1}$$

where $(\varepsilon_{1,t}, \varepsilon_{2,t})'$ is normally distributed with mean zero and covariance matrix Σ_t and ε_{1,t_1} and ε_{2,t_1} are independent of s_{1,t_2} and s_{2,t_2} for all t_1 and t_2.

The specification of the state variables $s_{1,t}$ and $s_{2,t}$ is based on the idea that the coincident and leading indicators share the same business cycle, but with the cycle of the CLI obviously leading the cycle of the CCI. Furthermore, stylized facts show that on average the CLI has a longer lead time when entering a recession than when entering an expansion. Hence, we allow for a different lead time when entering a recession than when entering an expansion. More formally, we assume that $s_{1,t}$ is a first-order two-state homogeneous Markov process with transition probabilities

$$\Pr[s_{1,t} = 0|s_{1,t-1} = 0] = p \quad \text{and} \quad \Pr[s_{1,t} = 1|s_{1,t-1} = 1] = q. \tag{2}$$

We specify $s_{2,t}$ such that it leads $s_{1,t}$ by κ_1 periods at peaks and by κ_2 periods at troughs, by defining $s_{2,t}$ as

$$s_{2,t} = \begin{cases} \prod_{i=\kappa_1}^{\kappa_2} s_{1,t+i} & \text{if } \kappa_1 \leq \kappa_2 \\ 1 - \prod_{i=\kappa_2}^{\kappa_1} (1 - s_{1,t+i}) & \text{if } \kappa_1 > \kappa_2. \end{cases} \tag{3}$$

To understand that this specification indeed gives rise to the desired asymmetric lead times, consider the case where $\kappa_1 \leq \kappa_2$. Defining $s_{2,t}$ as the product from $s_{1,t+\kappa_1}$ to $s_{1,t+\kappa_2}$, essentially implies that recessions in $y_{2,t}$ start κ_1 periods before recessions in $y_{1,t}$, while they end κ_2 periods earlier. Note that, consequently, recessions in $y_{2,t}$ are $|\kappa_2 - \kappa_1|$ periods shorter than recessions in $y_{1,t}$. On the other hand, if $\kappa_1 > \kappa_2$, recessions in $y_{2,t}$ are $|\kappa_1 - \kappa_2|$ periods longer than recessions in $y_{1,t}$. Obviously, lengthening the recession periods is equivalent to shortening the expansions. For that reason we define $s_{2,t}$ in (3) in terms of the product over $(1 - s_{1,t})$ in this case. An important feature of the model is that the lead times κ_1 and κ_2 are not fixed *a priori* but are treated as unknown parameters to be estimated.

A prominent feature of many US macro-economic time series is a large and persistent decline in volatility since the mid-1980s, see McConnell and Perez-Quiros (2000) and Sensier and van Dijk (2004), among others. To accommodate the effects of this 'Great Moderation' we allow for a single structural break in the covariance matrix of the shocks $(\varepsilon_{1,t}, \varepsilon_{2,t})'$:

$$\Sigma_t = \begin{cases} \Omega_0 & \text{if } t < \tau \\ \Omega_1 & \text{if } t \geq \tau, \end{cases} \tag{4}$$

where Ω_0 and Ω_1 are (2×2) covariance matrices. We treat the break date τ as an unknown parameter to be estimated.

The model parameters to be estimated are the regime-specific means $\mu_{j,s_{j,t}}$ and autoregressive parameters $\phi_{i,j}$ in (1), the transition probabilities p and q in (2), the lead-times κ_1 and κ_2 of $s_{2,t}$ in (3), and the elements of the covariance matrices Ω_1 and Ω_2 and the break date τ in (4). We adopt a Bayesian approach for parameter estimation. This is essential as we want to conduct inference on the discrete lead/lag time parameters κ_1 and κ_2, as well as the break date τ, which makes a frequentist approach infeasible. We use a prior specification that is relatively uninformative compared to the information in the likelihood. To obtain posterior results we employ the Gibbs sampling algorithm of Geman and Geman (1984) together with the data augmentation method of Tanner and Wong (1987). The unobserved state variables $\{s_{1,t}\}_{t=1}^{T}$ and $\{s_{2,t}\}_{t=1}^{T}$ are simulated alongside the model parameters, see Kim and Nelson (1999), among others.

References

Chauvet, M. and J. D. Hamilton (2006), Dating Business Cycle Turning Points, in C. Milas, P. Rothman, and D. van Dijk (eds.), *Nonlinear Time Series Analysis of Business Cycles*, Elsevier, North-Holland, pp. 1–54.

Chauvet, M. and J. Piger (2008), A Comparison of the Real-Time Performance of Business Cycle Dating Methods, *Journal of Business & Economic Statistics*, 26, 42–49.

Geman, S. and D. Geman (1984), Stochastic Relaxations, Gibbs Distributions, and the Bayesian Restoration of Images, *IEEE Transaction on Pattern Analysis and Machine Intelligence*, 6, 721–741.

Hamilton, J. D. (1989), A New Approach to the Econometric Analysis of Nonstationary Time Series and Business Cycles, *Econometrica*, 57, 357–384.

Kim, C.-J. and C. R. Nelson (1999), *State-Space Models with Regime Switching: Classical and Gibbs-Sampling Approaches with Applications*, MIT Press, Cambridge, MA.

McConnell, M. and G. Perez-Quiros (2000), Output Fluctuations in the United States: What Has Changed since the 1980s?, *American Economic Review*, 90, 1464–1476.

Paap, R., R. Segers, and D. van Dijk (2008), Do Leading Indicators Lead Peaks More Than Troughs?, *Journal of Business & Economic Statistics*, in press.

Sensier, M. and D. van Dijk (2004), Testing for Volatility Changes in US Macroeconomic Time Series, *Review of Economics and Statistics*, 86, 833–839.

Tanner, M. A. and W. H. Wong (1987), The Calculation of Posterior Distributions by Data Augmentation, *Journal of the American Statistical Association*, 82, 528–550.

The Conference Board (2001), *Business Cycle Indicators Handbook*, The Conference Board, New York.